1. The nodal coordinates of a three-node bar element are (x1,y1) = (−1,−1),(x2,y2) = (1,1) and (x3,y3) = (0,1). At the endof a finite element analysis, the axial stresses calculated at the three integration points were:σ1= 5kPa,σ2= 10kPa andσ3= 15kPa. Compute the equivalent nodal loads.

C = [-1 -1; 1 1; 0 1];

q = 3;

sig = [5; 10; 15];

nnodes = size(C, 1);

for i = 1: nnodes

q = quadrature(nnodes);

xi = q(i, 1);

[dN, n] = lin3\_derivs(xi);

N(i,:) = n';

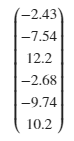
end

q = size(q,1);

sig\_nodal = N\sig;

F = compute\_F(C, sig, q);

vpa(F, 3)



2. Recover the nodal stresses for the last exercise.

vpa(sig\_nodal, 3)



function F = compute\_F(C, sig, q)

Q = quadrature(q);

nodeselement= size(C, 1);

ndof = 2;

numelement = 1;

F = zeros(nodeselement\*ndof,1);

C = C;

for i = 1: q

xi = Q (i, 1);

w = Q (i, 2);

B = compute\_B(C, q, ndof, xi);

[dN, N] = lin3\_derivs(xi);

J = C'\*dN;

F = F + B'\*sig(i)\*norm(J)\*w;

end

end

function B = compute\_B(C, q, ndof, xi)

q = quadrature(q);

Npst = size(q, 1);

nnodes = size(C, 1);

[dN, N] = lin\_shape\_form(nnodes, xi);

J = C'\*dN;

dNdX = dN\*pinv(J);

for j = 1: nnodes

c = (j-1) \* ndof;

B(1, c+1) = dNdX(j,1);

B(1, c+2) = dNdX(j,2);

end

end

function Q = quadrature(q)

quadrature\_1\_pts = [0.0 2.0];

quadrature\_2\_pts = [

-0.577350269189626 1.0

0.577350269189626 1.0];

quadrature\_3\_pts = [

-0.774596669241483 5.0/9.0

0.0 8.0/9.0

0.774596669241483 5.0/9.0];

if q == 1

Q = quadrature\_1\_pts;

elseif q == 2

Q = quadrature\_2\_pts;

elseif q == 3

Q = quadrature\_3\_pts;

else

Q = 0;

end

end

3. The nodal coordinates of a four-node quadrilateral element are (x1,y1) = (0,0), (x2,y2) = (2,0), (x3,y3) = (2,2) and (x4,y4) = (0,2). The displacements vector for this element is shown below. Assuming full integration, find the strain and the stress vector at each integration points. Later, find the internal forces vector F(e) (nodal forces). Use E= 20GPa and ν= 0.1.U(e)=[0.0 0.0 0.001 0.0 0.002 −0.001 −0.001 −0.001]T

Material

E = 20e6;

nu = 0.1;

q = 4;

type = 4;

D = E/(1-nu^2)\*[1 nu 0;

nu 1 0;

0 0 (1-nu)/2];

Malha

nos = [0 0; 2 0; 2 2; 0 2];

elem = [1 2 3 4];

coor = zeros(1, 8);

k=1;

for i = [linspace(1, 7, 4)]

coor(:,[i i+1]) = [elem(:, k)\*2-1, elem(:, k)\*2];

k=k+1;

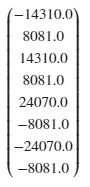
end

Deslocamentos sem restição

U = [0.0, 0.0, 0.001, 0.0, 0.002, -0.001, -0.001, -0.001]';

[f, N] = compute\_F(nos, D, U, q);

F = vpa(f, 4)



4. In the last exercise, recover the stress values at nodal points.

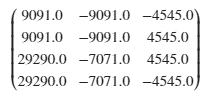
[sig] = stress\_strain(nos, elem, U, D, q, coor);

sig\_x = N\sig(:,:,1)';

sig\_y = N\sig(:,:,2)';

sig\_xy = N\sig(:,:,3)';

disp(vpa([sig\_x sig\_y sig\_xy], 4))



function [F, N] = compute\_F(C, D, U, Q)

q = quadrature(Q);

npts = size(q, 1);

nnodes = size(C, 1);

ndof = 2;

F = zeros(nnodes\*ndof, 1);

for i = 1:npts

xi = q (i, 1);

eta = q (i, 2);

w = q (i, 3);

B = compute\_B(C, Q, nnodes, ndof);

sig = D\*B(:,:,i)\*U;

[dN, n] = quad\_shape\_form(nnodes, xi, eta);

J = C'\*dN;

F = F + B(:,:,i)'\*sig\*det(J)\*w;

N(i,:) = n';

end

end

function [sig, eps] = stress\_strain(nodes,...

element,displacement,D,q,coordinateelem)

nodeselement= size(element, 2);

ndof = size(nodes, 2);

numelement = size(element, 1);

sig = zeros(numelement, q, 3);

eps = zeros(numelement, q, 3);

for i = 1: numelement

C = [nodes(element(i,:), :)];

U = displacement(coordinateelem(i,:));

for j = 1: q

B = compute\_B(C, q, nodeselement, ndof);

sig(i,j,:) = D\*B(:,:,j)\*U;

eps(i,j,:) = B(:,:,j)\*U;

end

end

end

function B = compute\_B(C, q, NoElem, ndof)

q = quadrature(q);

Npst = size(q, 1);

nnodes = size(C, 1);

B = zeros(3,nnodes\*2);

for i = 1:Npst

xi = q (i, 1);

eta = q (i, 2);

w = q (i, 3);

[dN, N] = quad\_shape\_form(NoElem, xi, eta);

J = C'\*dN;

dNdX = dN/J;

for j = 1: nnodes

c = (j-1) \* ndof;

b(1, c+1) = dNdX(j,1);

b(2, c+2) = dNdX(j,2);

b(3, c+1) = dNdX(j,2);

b(3, c+2) = dNdX(j,1);

end

B(:,:,i) = b;

end

end